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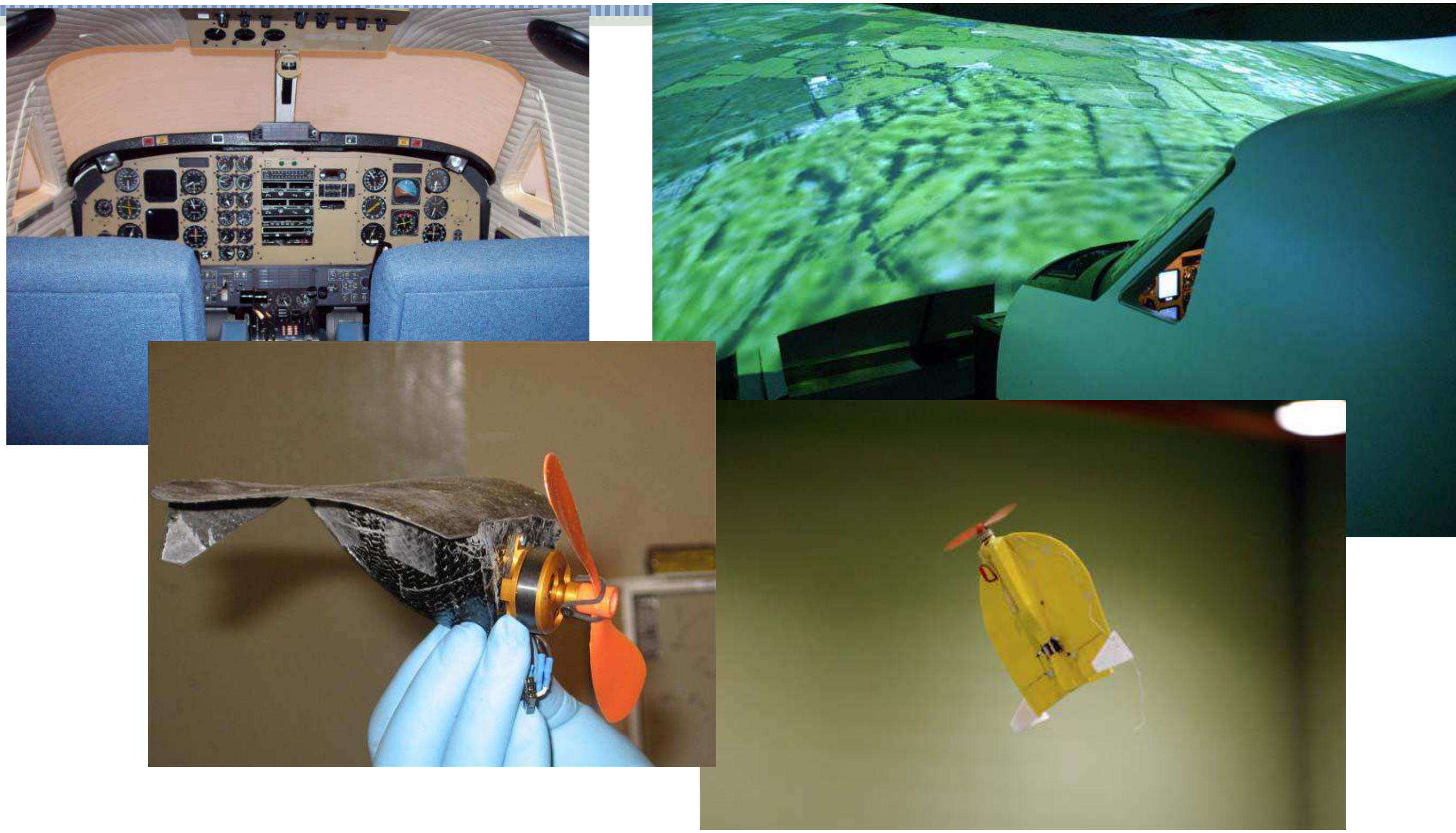

Networked Control Systems: A Multirate Wireless Perspective

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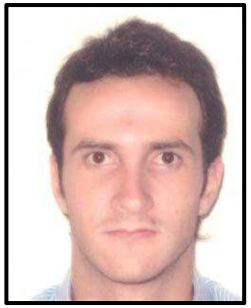

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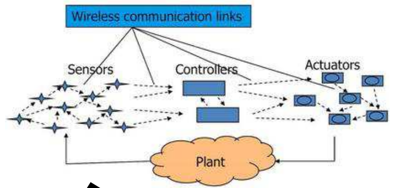
Autonomous vehicles

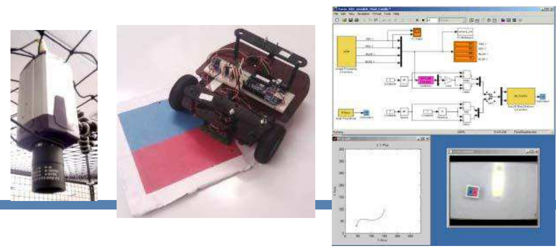
Fly-by-wireless

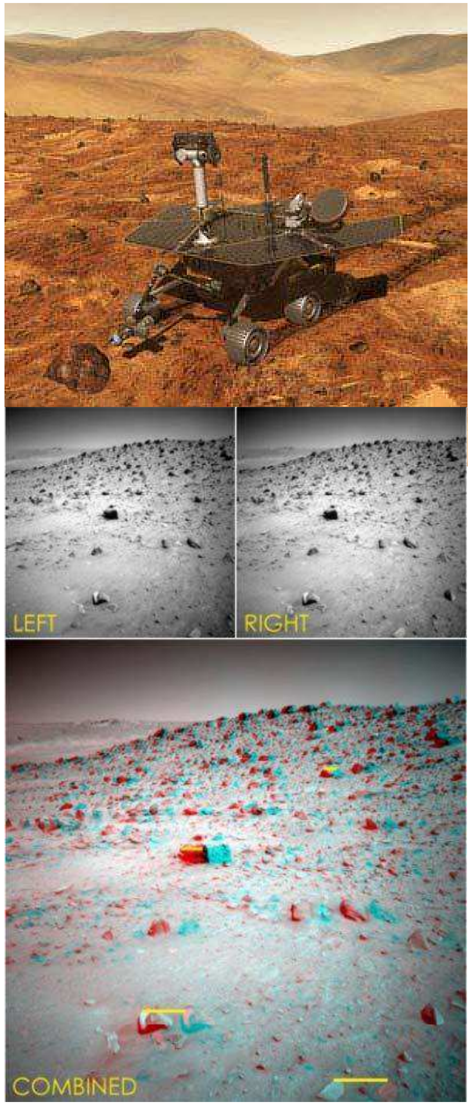
Multirate/Networked control



HYCONS Quadrotor

HYCONS Rover

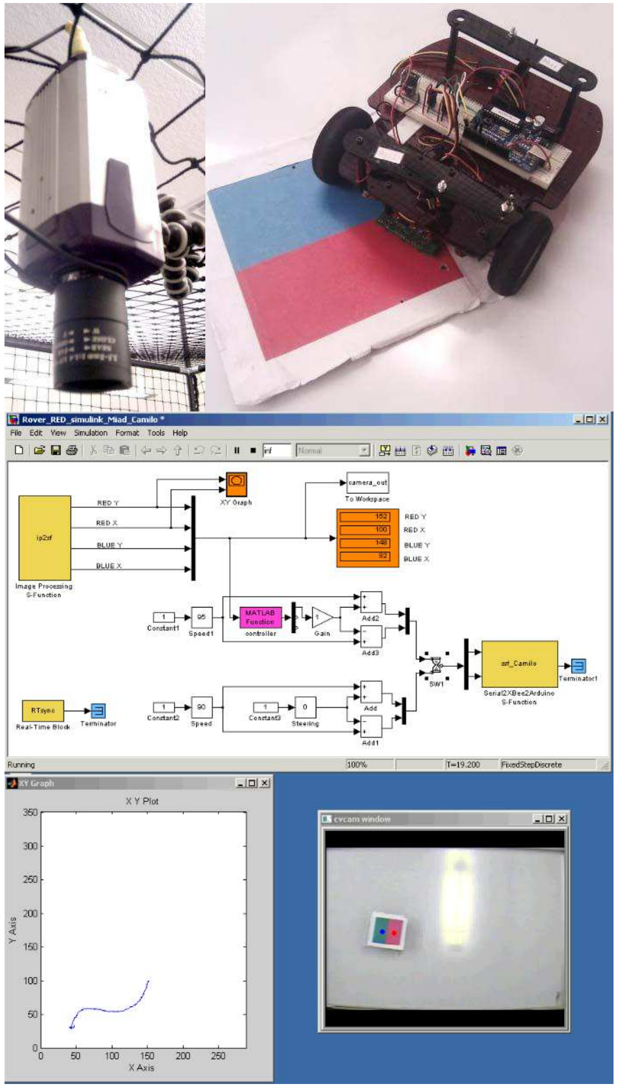




Mars Rovers (Spirit and Opportunity)

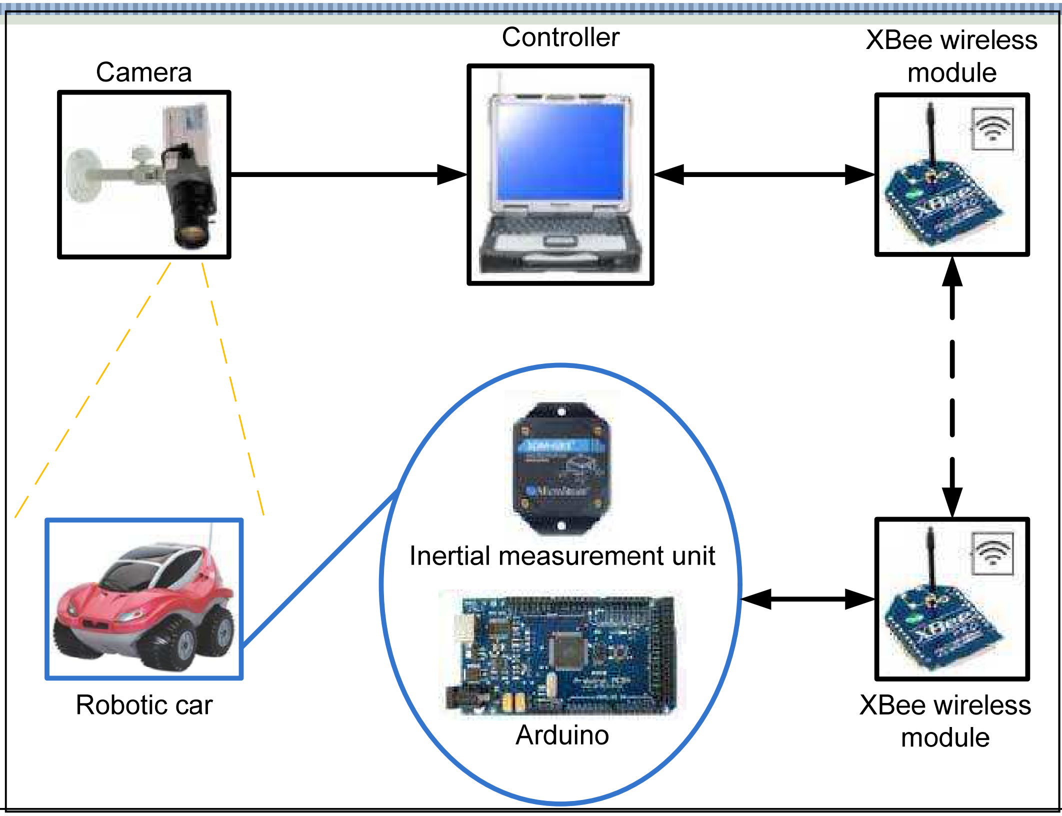
- There is no GPS in Mars
- Position estimation is based on Machine Vision systems
  - Visual Target Tracking (VTT)
    - Takes 0.5 to 1 minute to update
  - Visodom
    - Takes 1.5 to 2 minutes to update*
- Radio signals between rover and earth can take up to 30min.**

*[W. Kim, et. Al. "Visual Target Tracking on the Mars Exploration Rovers", International Symposium on Artificial Intelligence, Robotics and Automation in Space, CA, USA, 2008.]
**[Kim Ward, Project director, RAL Space. Interview for The Daily Mail, 2011]



HYCONS Rover

- Position estimation is based on a Machine Vision system:
  - It uses OpenCV library (open source) compiled for Matlab/Simulink
  - Different sampling rates can be emulated to test our controllers
- Wireless networked control using IEEE 802.15.4.
  - Different conditions for wireless signals can be emulated delays, package dropout, low latency, among others.



continuous-time model

ignore the network effects ⚠

continuous-time controller synthesis

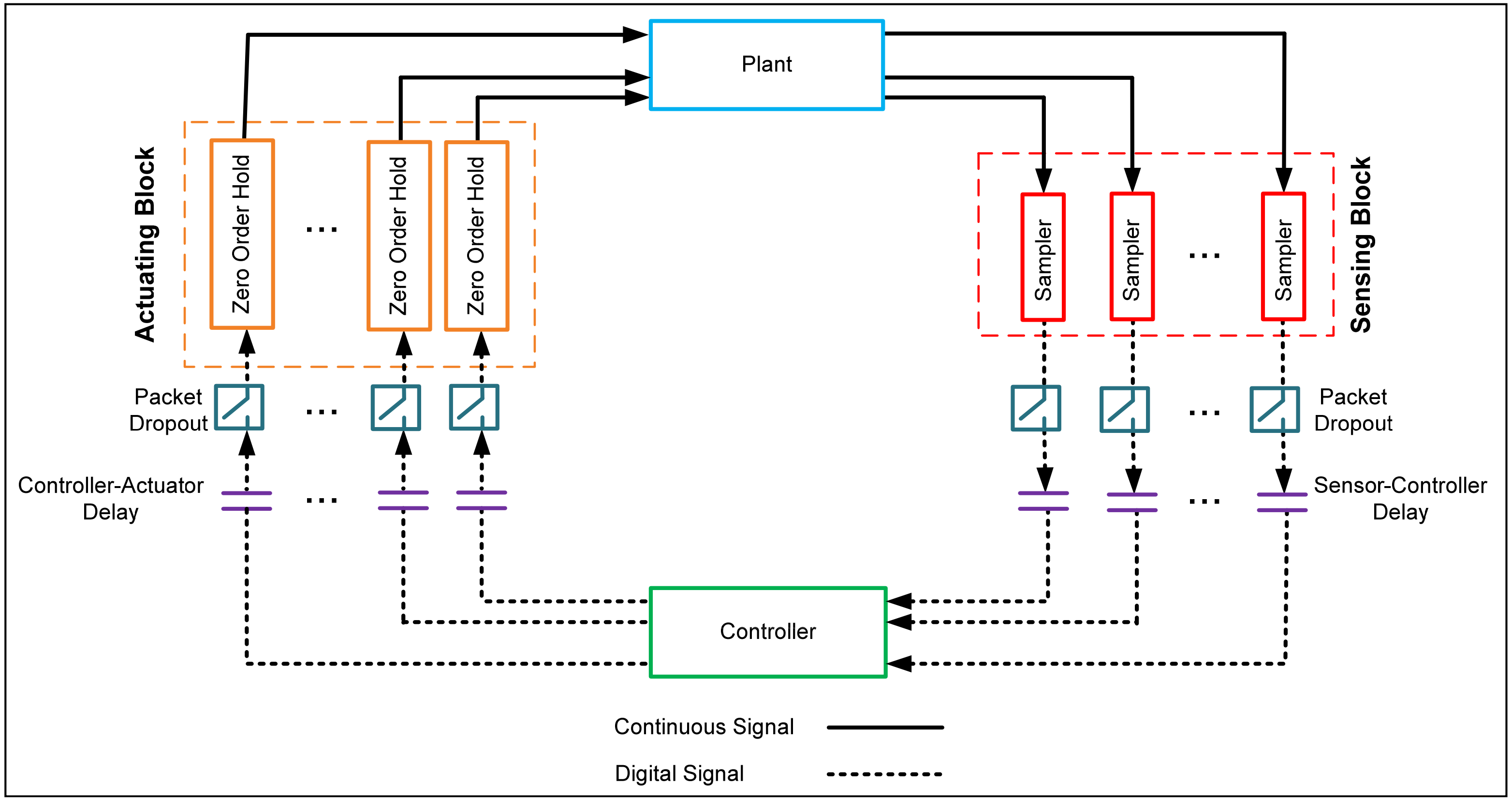
- multiple sensors with non-uniform sampling rates
- uncertain time-varying communication delays
- possibility of data packet dropouts
- D/A convertors with asynchronous updates

Outline

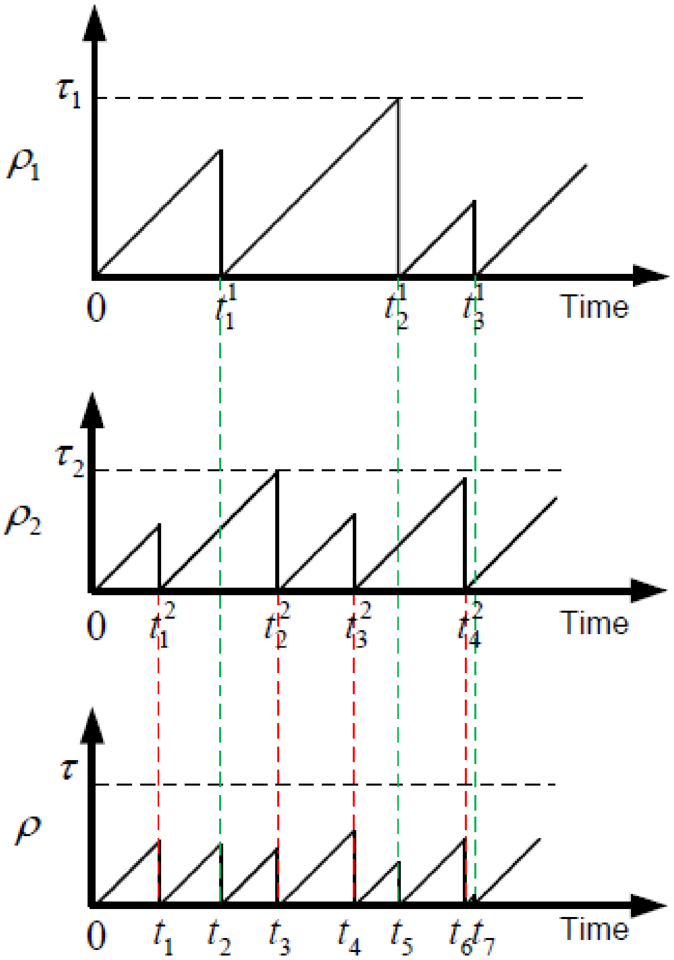
- Linear multirate networked systems
- Piecewise-affine systems
- Sampled-data piecewise-affine systems
- Applications
- Conclusions

- Multirate L. Sys. started in the 1950's using Z-transform methods [35]
- The concept of lifting operators was introduced in [36] and further developed in [37] by studying the error induced by quantization
- A method to simplify the characteristic equation of multirate control systems and the first design techniques were presented in [38]
- Recent results: Techniques to eliminate ripple [41], applications to Internet-based control [43], tracking control based on multirate feedforward control with generalizations of sampling periods [42].
- Recent applications: Visual servoing manipulators [44], radial control tracking in high-speed DVD players [45]
- Multirate using lifting operators only in simulation for attitude [16]

Networked Control System



Samplers are a source of delay



$$\rho_1(t) = t - t_n^1,\ t_n^1 \le t < t_{n+1}^1$$

$$0 \le \rho_1 < \tau_1$$

$$\rho_2(t) = t - t_n^2,\ t_n^2 \le t < t_{n+1}^2$$

$$0 \le \rho_2 < \tau_2$$

$$\rho(t) = \min\{\rho_1, \rho_2\} = t - t_n,\ t_n \le t < t_{n+1}$$

$$0 \le \rho < \tau = \min\{\tau_1, \tau_2\}$$

Linear multi-rate sampled model

- Continuous-time system:

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$u(t) = Kx(t)$$

- Sampled-data system:

$$\dot{x}(t) = Ax(t) + Bu(t)$$
$$u(t) = K\overline{x}(t) = \sum_{i=1}^{m} K_i x_i(t - \rho_i(t))$$

where $\overline{x}(t) = \begin{bmatrix} x_1(t - \rho_1(t)) \\ \vdots \\ x_m(t - \rho_m(t)) \end{bmatrix}$

and m is the number of sensors

Theorem

The closed-loop system is exponentially stable if there exist $P > 0$, $R > 0$, $\overline{R} > 0$, symmetric matrix X, and matrices N and $\overline{N}$, with appropriate dimensions, satisfying

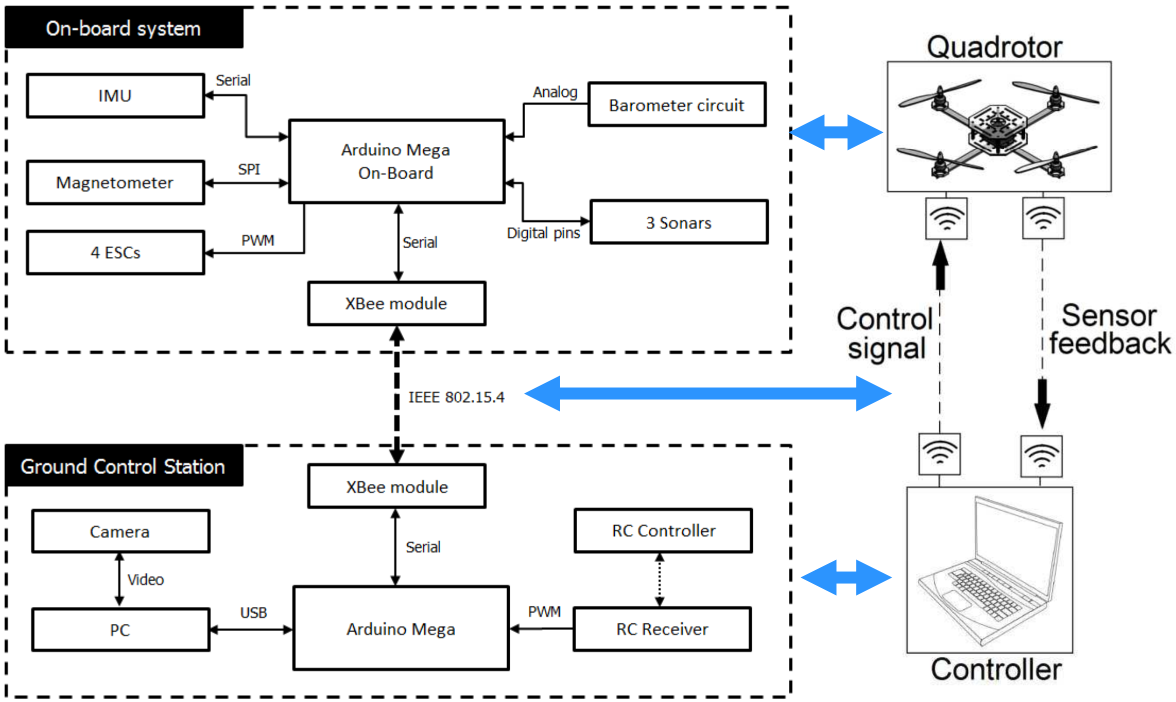
$$\begin{bmatrix} P & 0 \\ 0 & 0 \end{bmatrix} + \tau X > 0$$

$$\Psi + \tau M_1 < 0$$

$$\begin{bmatrix} \Psi + \tau M_2 & \tau N & \overline{N} \\ \tau N^T & -\tau R & 0 \\ \overline{N}^T & 0 & -\overline{R} \end{bmatrix} < 0$$

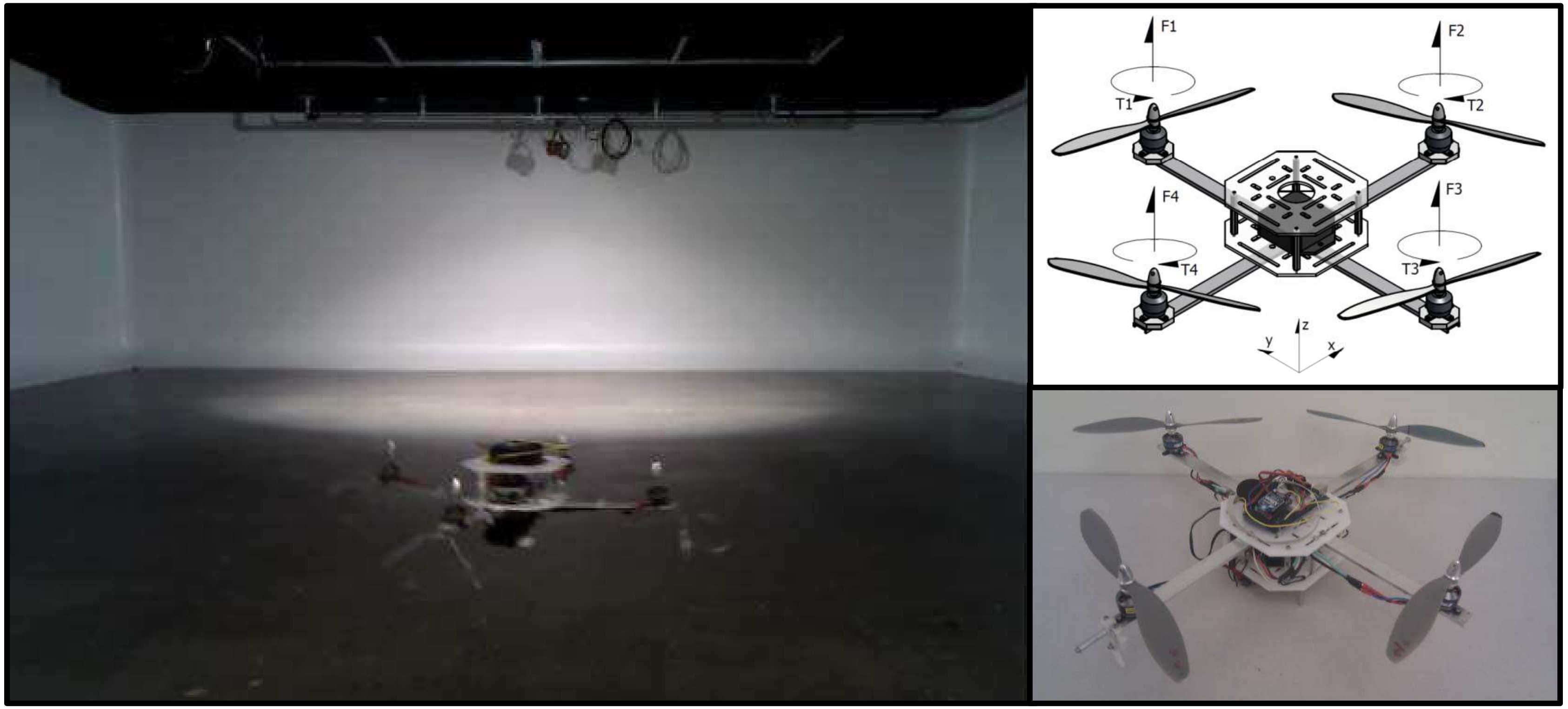
where Ψ, M_1, and M_2 are matrix functions of system parameters A, B, and K.

HYCONS Quadrotor



- Embedded and off-board controllers (through Real-Time Wireless Network) can be implemented.
- Modular design allows to add/replace sensors and components
- Different conditions can be created to study the influence on the system's stability when faults occur on the network, sensors or actuators.

HYCONS Quadrotor



Sampled-data linear systems

Lyapunov-Krasovskii-based stability conditions

Sampled-data piecewise affine systems

- [Azuma, Imura; 2003]

- [Rodrigues; 2007] Stability conditions for constant sampling intervals

- [Samadi, Rodrigues; 2009] Stability conditions for uncertain sampling intervals

- [This talk] Asymptotic stability conditions for uncertain sampling intervals

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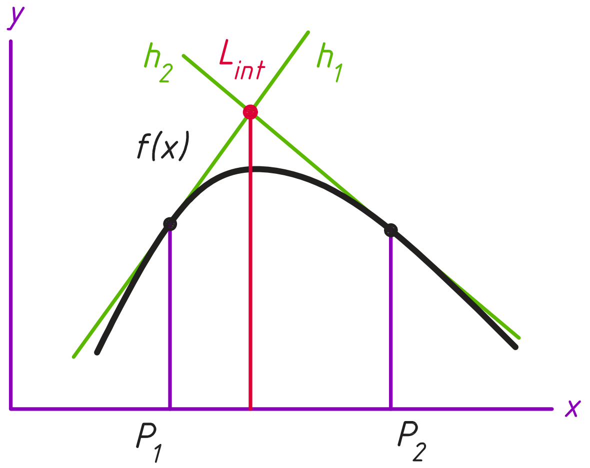
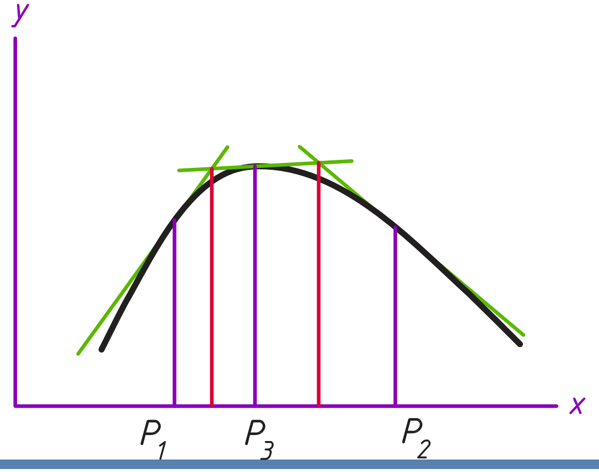
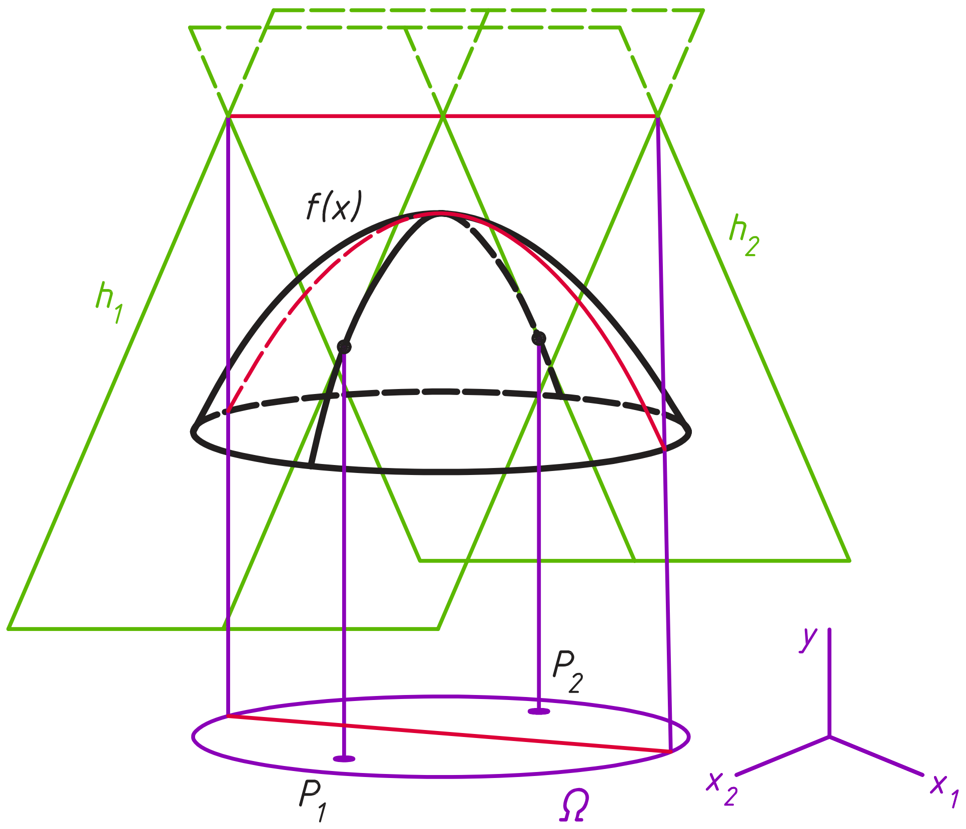
Piecewise Affine Modeling of Nonlinear Systems

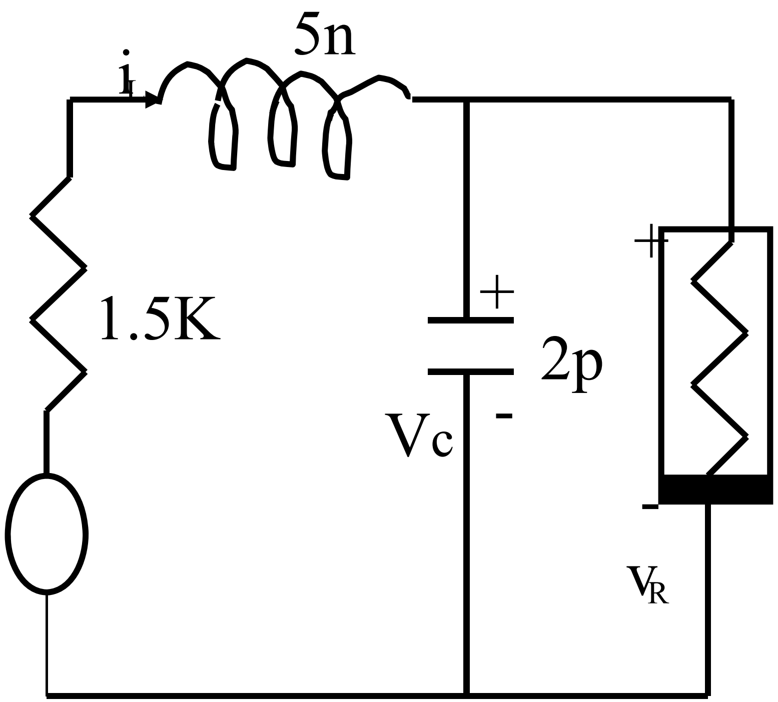
- Curves in 2D

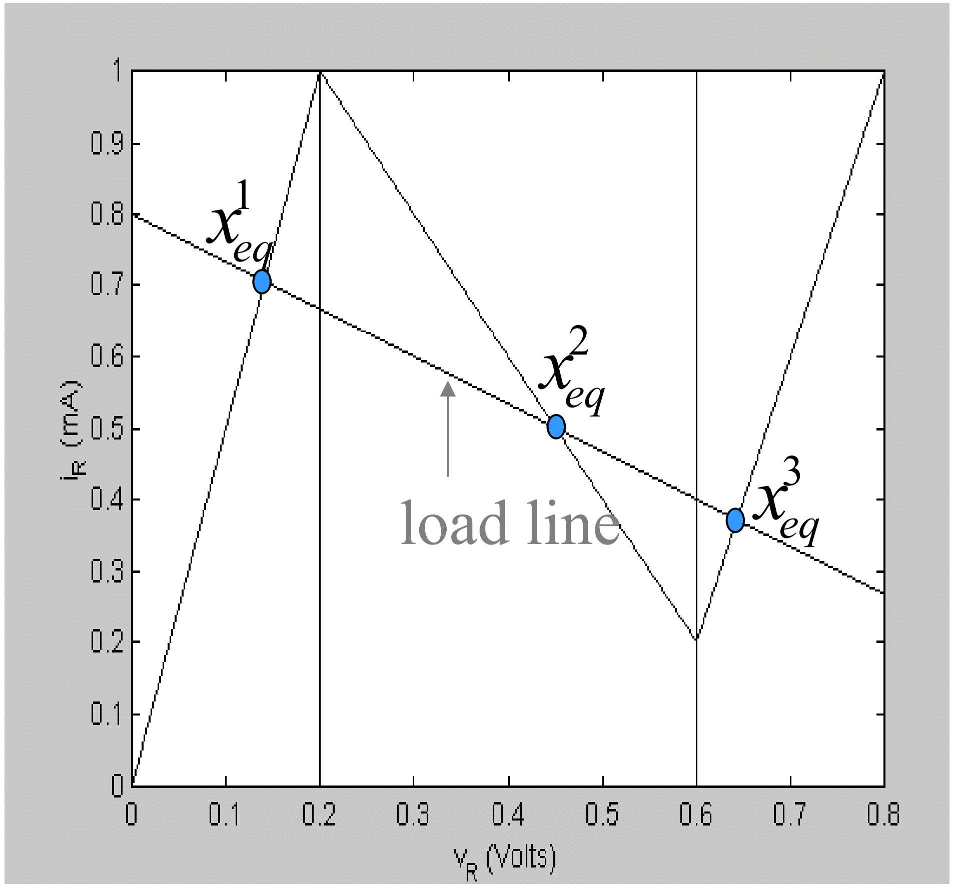




- Surfaces in 3D




5n
i
1.5K
+
2p
Vc
-
+
-
V R


1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
i R (mA)
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
v R (Volts)
x_{eq}^1
x_{eq}^2
x_{eq}^3
load line

- For the circuit we have

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -30 & -20 \\ 0.05 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 24 \\ -50 f(x_2) \end{bmatrix} + \begin{bmatrix} 20 \\ 0 \end{bmatrix} u$$

- After replacing the function f(.) by its expression we get

$$\dot{x} = A_i x + b_i + B_i u, \quad x \in R_i$$

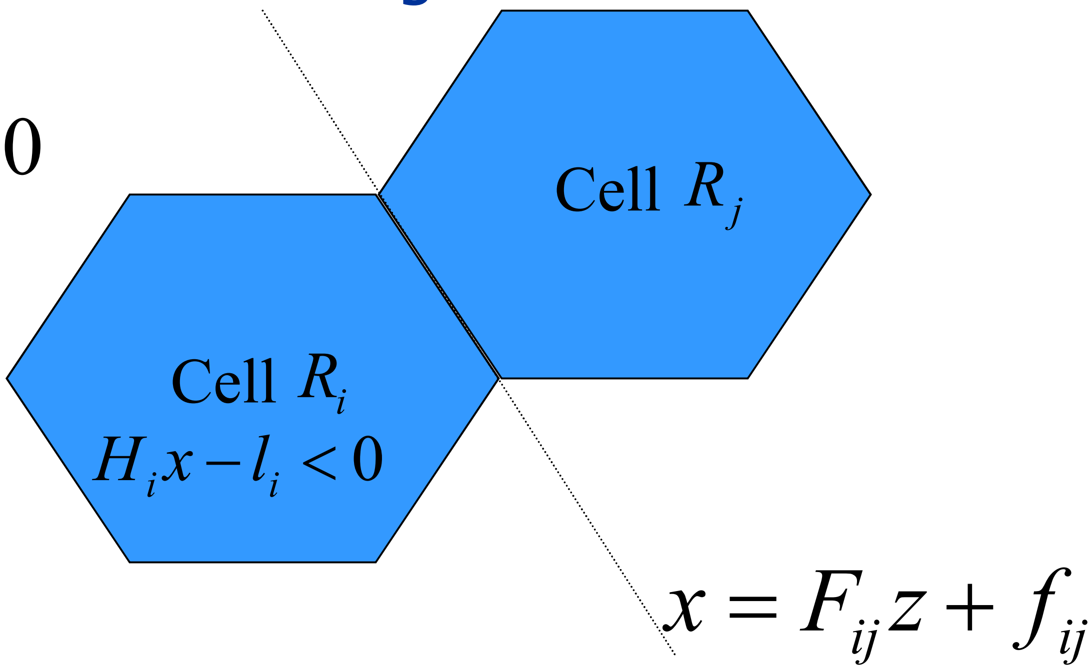
- Dynamics

$$\dot{x} = A_i x + b_i + B_i u, \quad x \in R_i, \quad f_i(x) = A_i x + b_i, \quad g_i(x) = B_i$$

$$y = C_i x + d_i$$

- Switching when

$$h_{ij}(x) = H_{ij}^T x + l_{ij} = 0$$



Piecewise Affine Controller Synthesis

- **Given a piecewise-affine system**

$$\dot{x} = A_i x + b_i + B_i u, \quad x \in R_i$$

find a stabilizing piecewise-affine state feedback control signal

$$u = K_i x + k_i, \quad x \in R_i$$

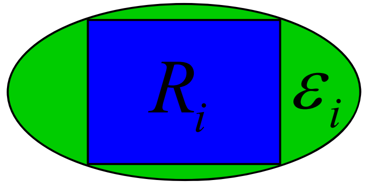
that exponentially stabilizes the closed-loop system to the origin and analyze its performance when the measurements are subject to norm bounded noise.

- **Quadratic Lyapunov function**

$$V = x^T P x$$

- **Assume R_i is contained in a union of ellipsoids of the form**

$$\varepsilon_{ij} = \left\{ x \mid \left\| E_{ij} x + f_{ij} \right\| \le 1 \right\}$$



$$V > 0,\ x \in R_i \Rightarrow \dot{V} < -\alpha V$$

$$P > 0, \quad \lambda_{ij} < 0$$

$$\begin{bmatrix} \overline{A}_i^T P + P\overline{A}_i + \alpha P + \lambda_{ij} E_{ij}^T E_{ij} & (P\overline{b}_i + \lambda_{ij} E_{ij}^T f_{ij}) \\ (.)^T & -\lambda_{ij}\left(1 - f_{ij}^T f_{ij}\right) \end{bmatrix} < 0$$

$$\overline{A}_i = A_i + B_i K_i, \quad \overline{b}_i = b_i + B_i k_i$$

- **For piecewise-affine slab systems the conditions are equivalent to (*)**

$$Q > 0, \quad \mu_i < 0$$

$$\begin{bmatrix} A_i Q + Q A_i^T + B_i Y_i + Y_i^T B_i^T + \alpha Q + \mu_i \bar{b}_i \bar{b}_i^T & (Q E_{ij}^T + \mu_i \bar{b}_i f_{ij}^T) \\ (.)^T & -\mu_i \left(I - f_{ij} f_{ij}^T \right) \end{bmatrix} < 0$$

$$Q = P^{-1}, \quad \mu_i = \lambda_i^{-1}, \quad \bar{b}_i = b_i + B_i k_i, \quad Y_i = K_i Q$$

where

$$\begin{aligned} &find \quad Q, Y_i, k_i, \mu_i \\ &s.t \quad (*), -l_0 < k_i < l_0 \end{aligned}$$

- **Algorithm 1 (Rodrigues, Boyd, Systems & Control Letters 2005)**
  1. **Grid the domain of** $k_i, i = 1, \cdots, N,$
  2. **For fixed** $\alpha \geq 0$ **, solve LMI for each point in the grid until feasible point is found**

- **Algorithm 2 (Rodrigues, Boyd, Systems & Control Letters 2005)**

  Trace Maximization Algorithm: iterative algorithm with convex relaxations. Yields exact solution when the obtained solution makes optimization functional equal to zero

- **Algorithm 3 (Samadi, Rodrigues, Automatica 2009)**

  Ignore term $\mu_i \overline{b}_i \overline{b}_i^T$ **because this term is negative semidefinite. Solve relaxed problem. The solution to relaxed problem is also a solution to the original problem**

- **Analysis under measurement error**

$$y = x + \eta, \quad \|\eta\| < N$$

- **Control Input**

$$u = K_j y + k_j, \quad y \in \Re_j$$

- **Closed Loop System**

new terms

$$\dot{x} = \left(A_i + B_i K_i + B_i \Delta K_{ij}\right) + b_i + B_i k_i + B_i \left(\Delta k_{ij} + K_j \eta\right)$$

$$\Delta K_{ij} = K_j - K_i, \quad \Delta k_{ij} = k_j - k_i$$

- **Theorem: The trajectories of the closed-loop system converge exponentially to the set**

$$\Omega = \left\{ x \mid V(x) \le \sigma_{\max}(P)\mu_\theta^2 \right\}$$

provided

$$\Delta K < \frac{\sigma_{\min}(P)\alpha\theta}{2\sigma_{\max}(P)B},$$

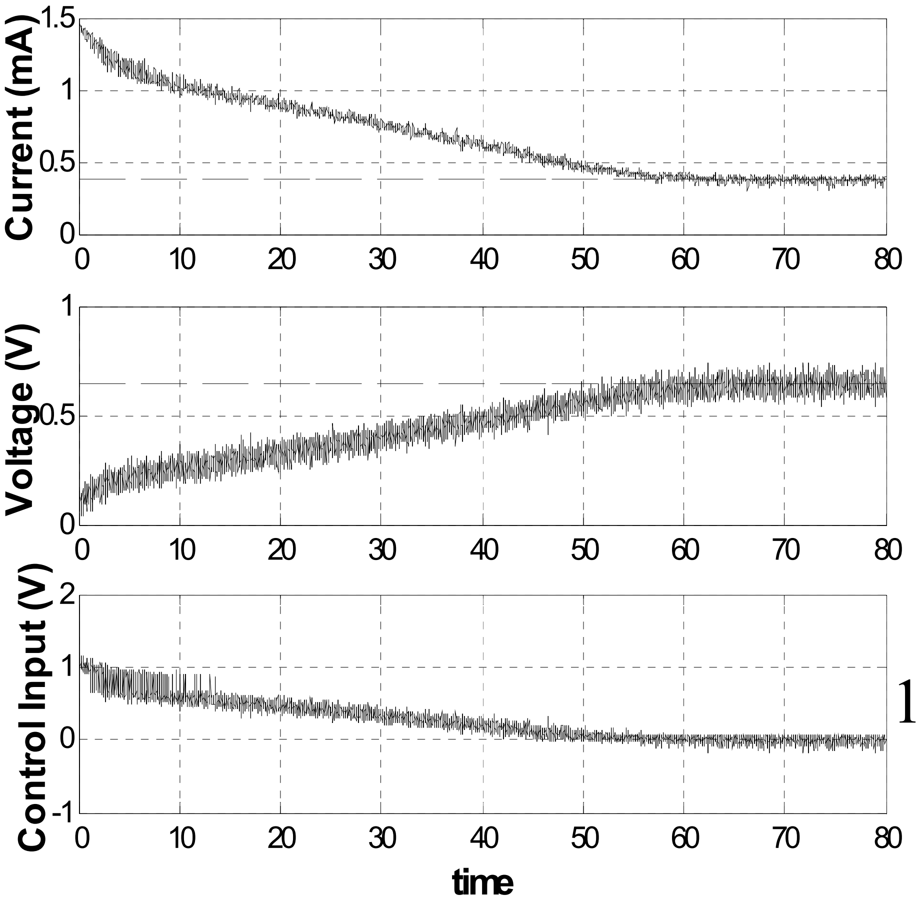
where

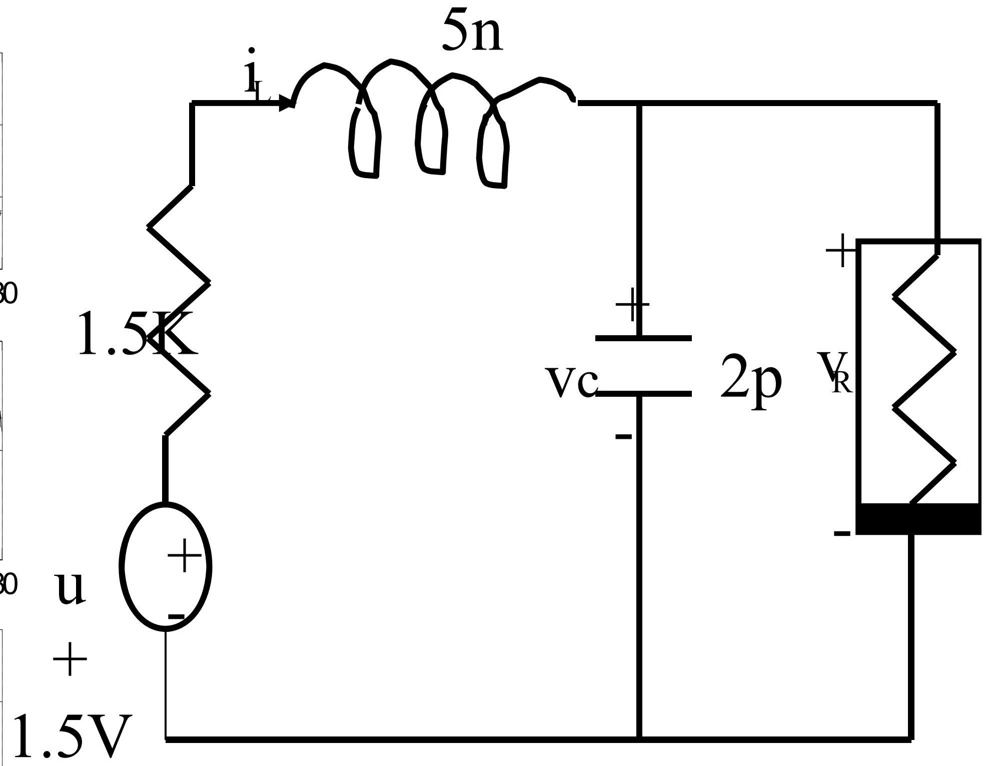
$$\mu_\theta = \frac{2\sigma_{\max}(P)B\left(KN + \Delta k\right)}{\alpha\theta\sigma_{\min}(P) - 2\sigma_{\max}(P)B\Delta K},$$

$$0 < \theta < 1,\ B = \max_i \left\| B_i \right\|,\ K = \max_i \left\| K_i \right\|,$$

$$\Delta K = \max_{i,j} \left\| \Delta K_{ij} \right\|, \Delta k = \max_{i,j} \left\| \Delta k_{ij} \right\|$$

- **Norm bounded noise (Gaussian noise passed through saturation block)**





- **Piecewise quadratic Lyapunov function:**

$$V(x)=\sum_{i=1}^{M}\beta_i(x)V_i(x),\quad \beta_i(x)=\begin{cases}1, & x\in R_i\\ 0, & x\in R_j,\ j\neq i\end{cases}$$

$$V_i(x)=\bar{x}^T\bar{P}_i\bar{x},\quad \bar{P}_i=\begin{bmatrix}P_i & q_i\\ q_i^T & r_i\end{bmatrix}$$

- **Feedback controller:**

$$u=K_ix+k_i=\bar{K}_i\bar{x},\quad \bar{x}=\begin{bmatrix}x & 1\end{bmatrix}^T,\quad x\in R_i$$

1. **Design a linear local controller in the region holding the equilibrium point for local performance.**

2. **Find a local quadratic Lyapunov function.**

3. **Minimize the upper bound of the difference between the closed-loop dynamics of all other regions and the reference model:**

$$\left\| \overline{A}_i + \overline{B}_i \overline{K}_i - (\overline{A}_{i^*} + \overline{B}_{i^*} \overline{K}_{i^*}) \right\| < \gamma$$

subject to the following constraints:

– **Continuity of the control input and Lyapunov function** $V(x) > 0 \quad \dot{V}(x) < 0$

–

Given x_{cl} fix $P_{i^\star}$, $K_{i^\star}$, $k_{i^\star}$ and solve

$$
\begin{aligned}
& \min(\beta) \\
\text{s.t.} \quad & \bar{P}_i \equiv \begin{bmatrix} P_i & -P_i x_{cl} \\ -x_{cl}^T P_i & r_i + x_{cl}^T x_{cl} \end{bmatrix} \\
& P_i = P_i^T > 0 \\
& \bar{F}_{ij}^T (\bar{P}_i - \bar{P}_j) \bar{F}_{ij} = 0, \text{for } j \in \mathcal{N}_i \\
& (\bar{K}_i - \bar{K}_j) \bar{F}_{ij} = 0, \text{for } j \in \mathcal{N}_i \\
& Z_i \succ 0 \\
& \bar{P}_i - \check{E}_i^T Z_i \check{E}_i > 0 \\
& \Lambda_i \succ 0 \\
& \bar{P}_i(\bar{A}_i + \bar{B}_i \bar{K}_i) + (\bar{A}_i + \bar{B}_i \bar{K}_i)^T \bar{P}_i + \check{E}_i^T \Lambda_i \check{E}_i < 0 \\
& -K_{Lim} \prec \bar{K}_i \prec K_{Lim} \\
& -\beta \prec (\bar{A}_i + \bar{B}_i \bar{K}_i) - (\bar{A}_{i^\star} + \bar{B}_{i^\star} \bar{K}_{i^\star}) \prec \beta \\
& \bar{K}_{i^\star} = [K_{i^\star} \quad k_{i^\star}] \\
& \bar{P}_{i^\star} = \begin{bmatrix} P_{i^\star} & -P_{i^\star} x_{cl} \\ -x_{cl}^T P_{i^\star} & x_{cl}^T x_{cl} \end{bmatrix} \\
& \text{for } i \in \mathcal{I} = \{1, \dots, M\}, i \neq i^\star
\end{aligned}
$$

- $\bar{F}_{ij}^T (\bar{P}_i - \bar{P}_j) \bar{F}_{ij} = 0$ → **V continuous**
- $(\bar{K}_i - \bar{K}_j) \bar{F}_{ij} = 0$ → **Continuity of control input**
- $\bar{P}_i - \check{E}_i^T Z_i \check{E}_i > 0$ → **Positive definiteness of $V_i(x)$**
- $\bar{P}_i(\bar{A}_i + \bar{B}_i \bar{K}_i) + \dots < 0$ → **Decrease over time**

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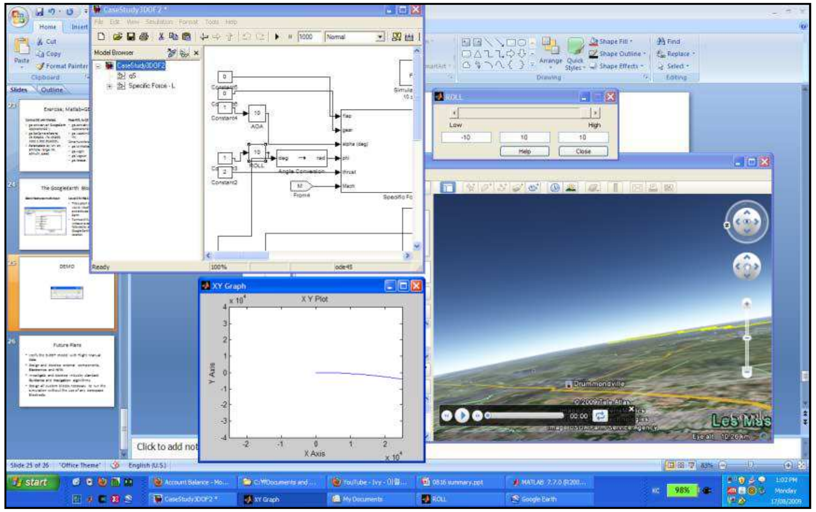
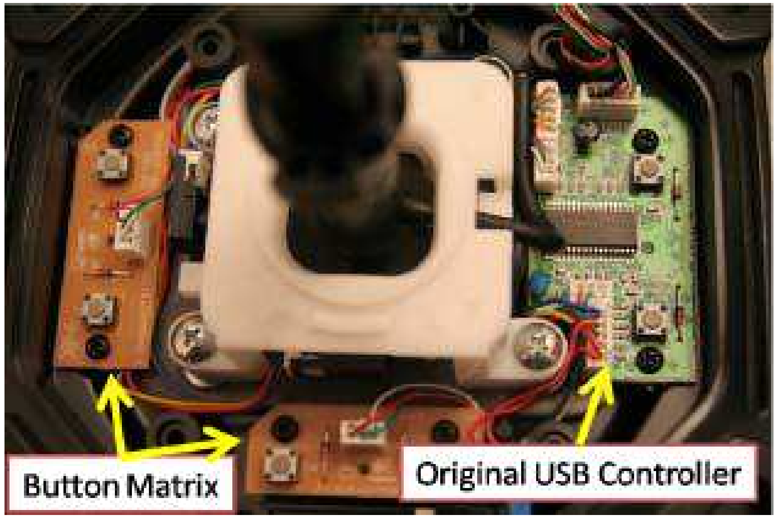
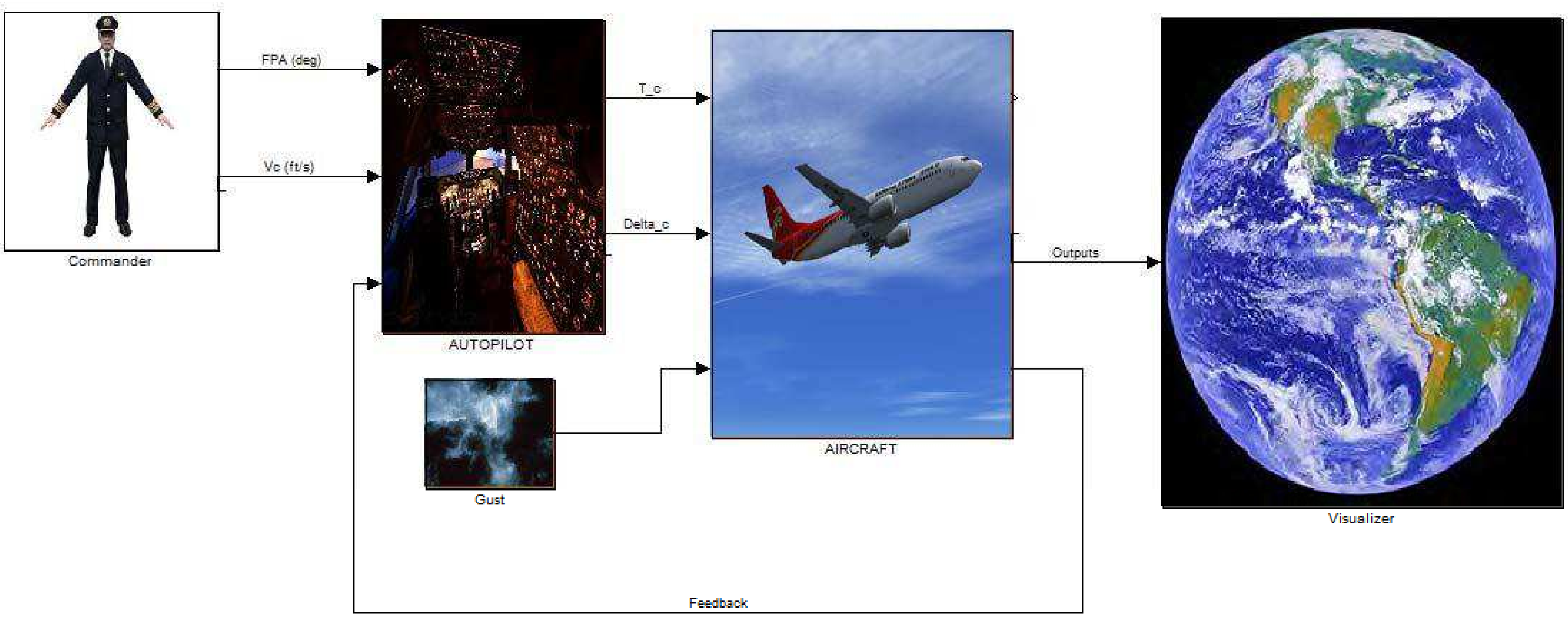
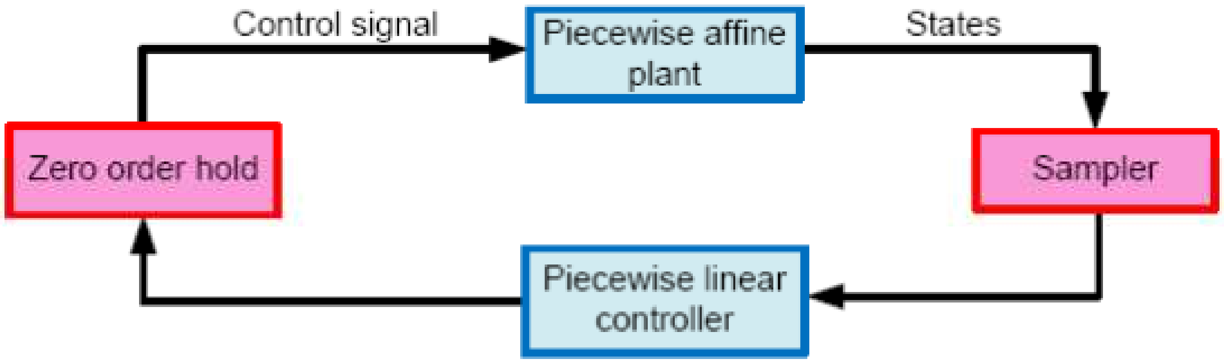




Matlab/Google Earth Simulator


FPA (deg)
Vc (ft/s)
Commander
T_c
Delta_c
AUTOPILOT
Gust
AIRCRAFT
Outputs
Visualizer
Feedback

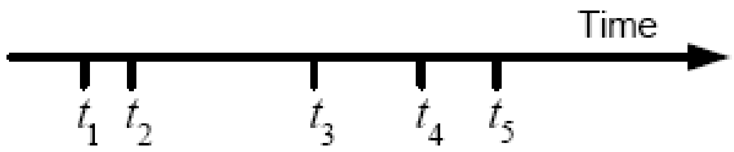


Objectives

- Sufficient conditions for asymptotic stability of sampled-data piecewise affine systems
- Optimization problem in terms of linear matrix inequalities

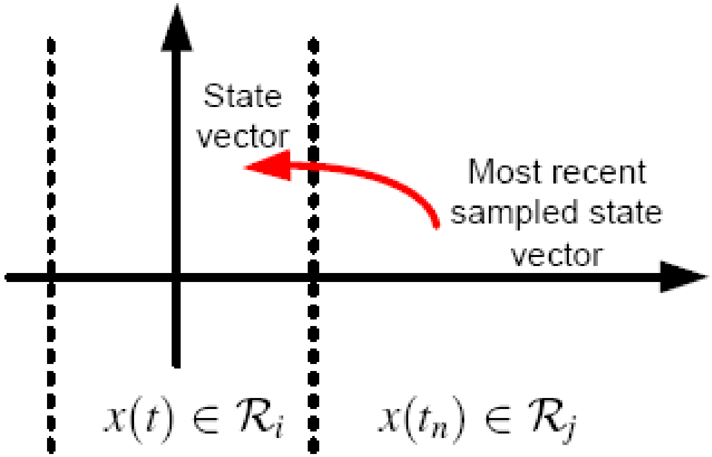
Challenge: time-varying delay

Uncertain sampling intervals



$$\epsilon = \inf_{n\in\mathbb{N}} (t_{n+1} - t_n) > 0 \quad \text{and} \quad \tau = \sup_{n\in\mathbb{N}} (t_{n+1} - t_n) > 0$$

State vector and its most recent sampling might be in different regions



$$\begin{aligned} \forall\, t \in [t_n, t_{n+1}) : \quad \dot{x}(t) &= A_i x(t) + a_i + BK_j x(t_n) \\ &= A_i x(t) + a_i + BK_i x(t_n) + Bw(t) \\ &\Downarrow \\ w(t) &= (K_j - K_i) x(t_n) \end{aligned}$$

$$||w(t)|| \leq \Delta K ||x(t_n)||, \;\; \Delta K = \max_{i,j} ||K_j - K_i||$$

Theorem

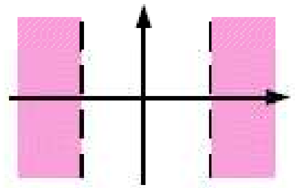
Consider a piecewise affine system in feedback with a sampled-data piecewise linear controller with sampling intervals larger than ϵ and smaller than τ. The closed-loop system is asymptotically stable if there exist $P > 0$, $R > 0$, and $X > 0$, symmetric matrices $\Lambda_i \succeq 0$, $i \notin I^*$, matrices N_i, $i \notin I^*$, and $\overline{N}_i$, $i \in I^*$, with appropriate dimensions, and $\gamma > 0$, $0 < \theta < 1$, and $\eta > 0$, satisfying

$$\Delta K^2 \gamma < \theta$$

- for all regions $\mathcal{R}_i$ that do not contain the origin

$$\Omega_i + \tau(M_{1i} + M_{2i}) + S_i < 0$$

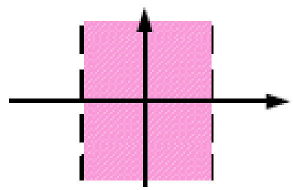
$$\begin{bmatrix} \Omega_i + \tau(M_{2i} + M_{3i}) + S_i & \tau N_i \\ \tau N_i^T & -\tau R \end{bmatrix} < 0$$



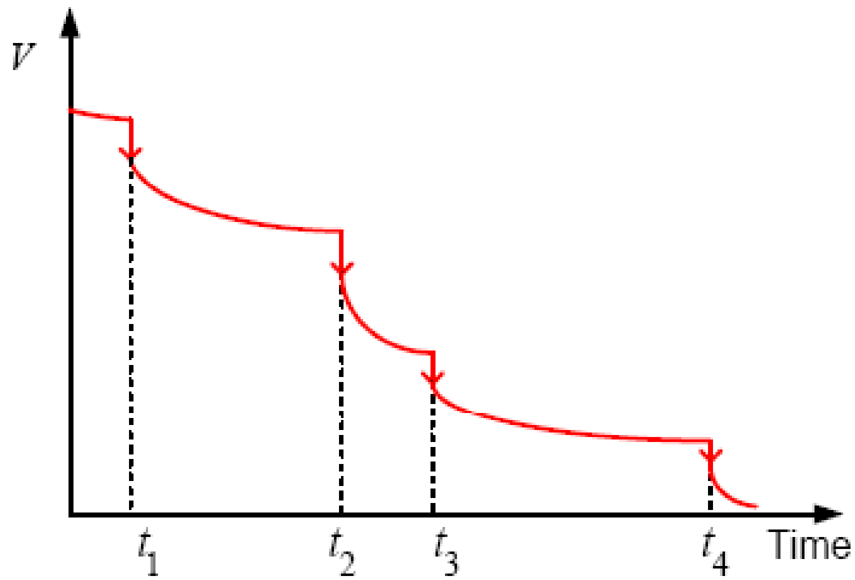
- for all regions $\mathcal{R}_i$ that contain the origin

$$\overline{\Omega}_i + \tau(\overline{M}_{1i} + \overline{M}_{2i}) < 0$$

$$\begin{bmatrix} \overline{\Omega}_i + \tau(\overline{M}_{2i} + \overline{M}_{3i}) & \tau \overline{N}_i \\ \tau \overline{N}_i^T & -\tau R \end{bmatrix} < 0$$



Idea of the proof



The inequalities in the Theorem $\Rightarrow \dot{V}(t) < 0,\ t \in (t_n, t_{n+1}),\ \forall n \in \mathbb{N}$

Maximizing the sampling time

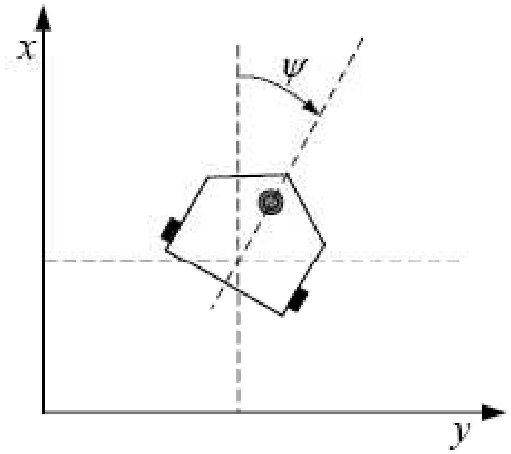
maximize τ

subject to $P > 0,\ R > 0,\ X > 0,\ \Lambda_i \succeq 0,$ for $i \notin I^*,$

$\gamma > 0,\ 0 < \theta < 1,\ \eta > 0,$

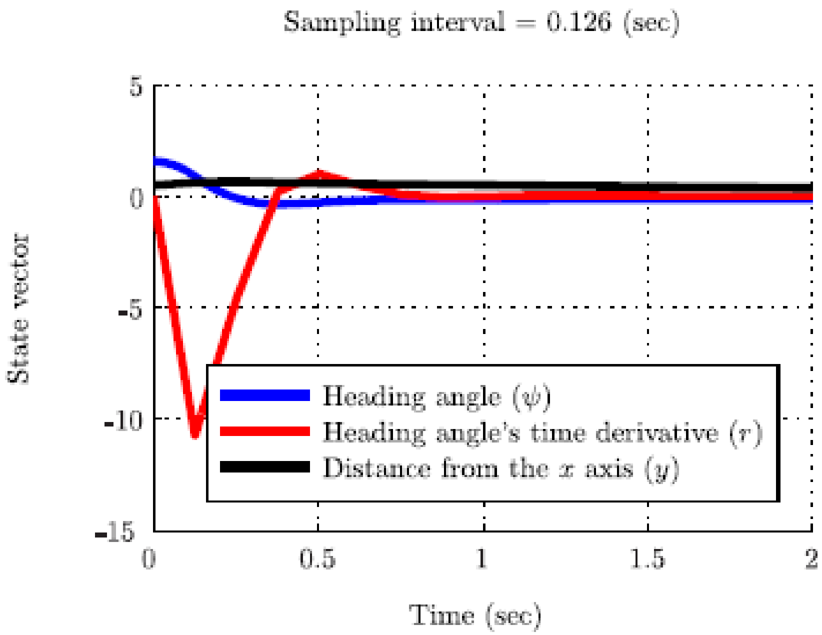
and the linear matrix inequalities in the Theorem

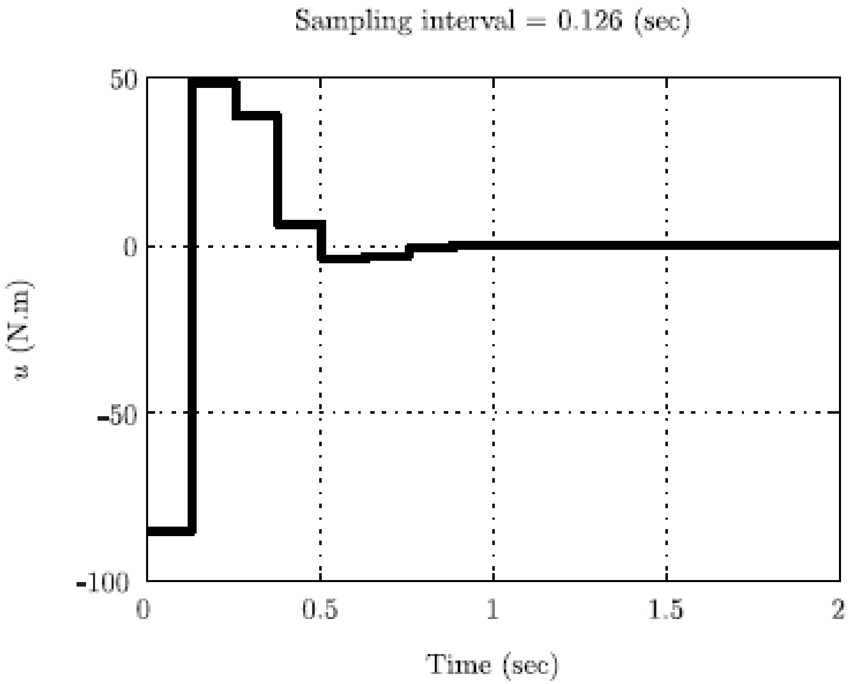
Example: Path Following



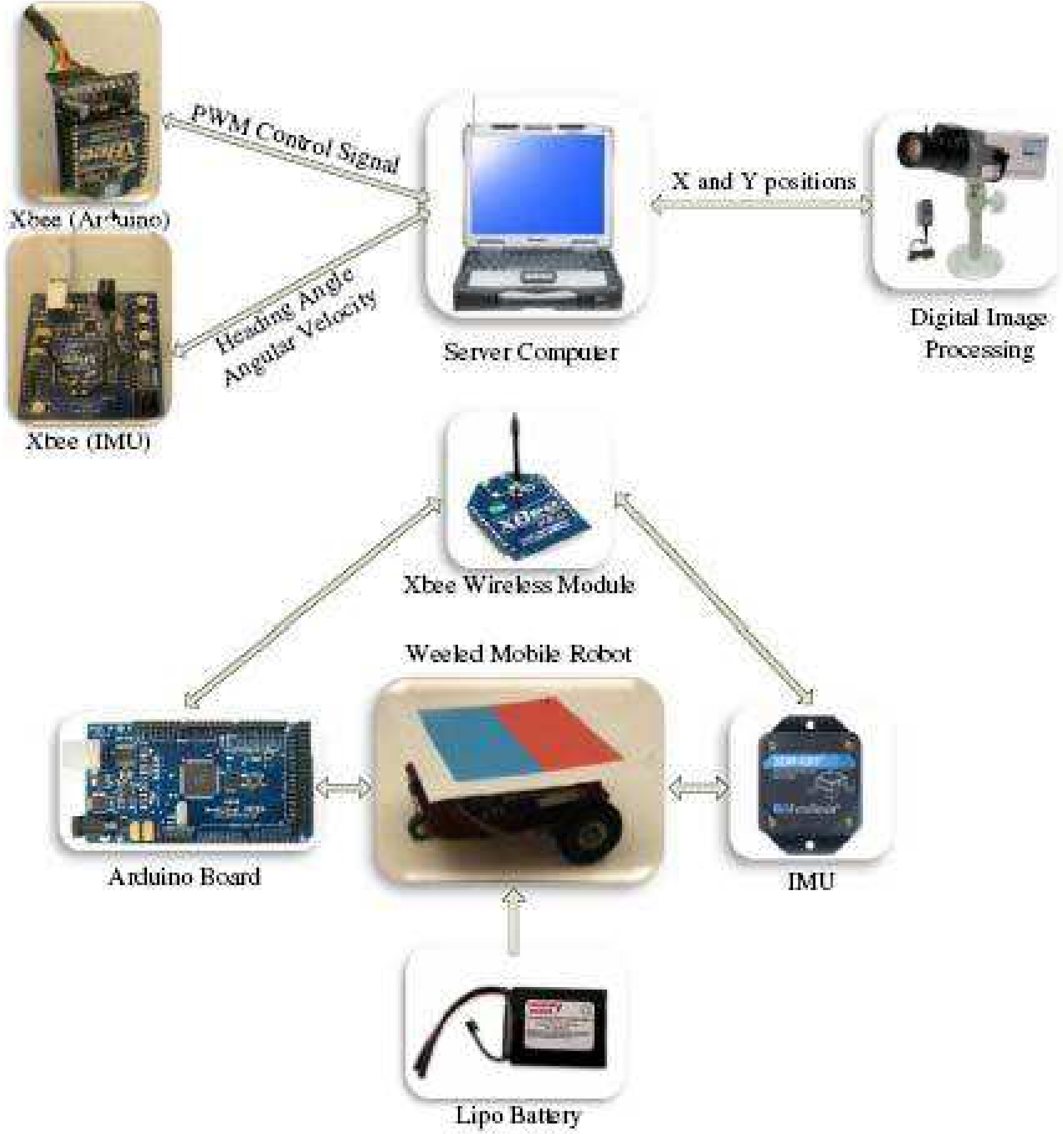
$$\begin{bmatrix}\dot{\psi}\\ \dot{r}\\ \dot{y}\end{bmatrix}=\begin{bmatrix}0 & 1 & 0\\ 0 & -k/I & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\psi\\ r\\ y\end{bmatrix}+\begin{bmatrix}0\\ 0\\ v\sin(\psi)\end{bmatrix}+\begin{bmatrix}0\\ 1/I\\ 0\end{bmatrix}u$$

$$k=0.01\ \text{(Nms)},\ I=1\ (\text{Kgm}^2)$$

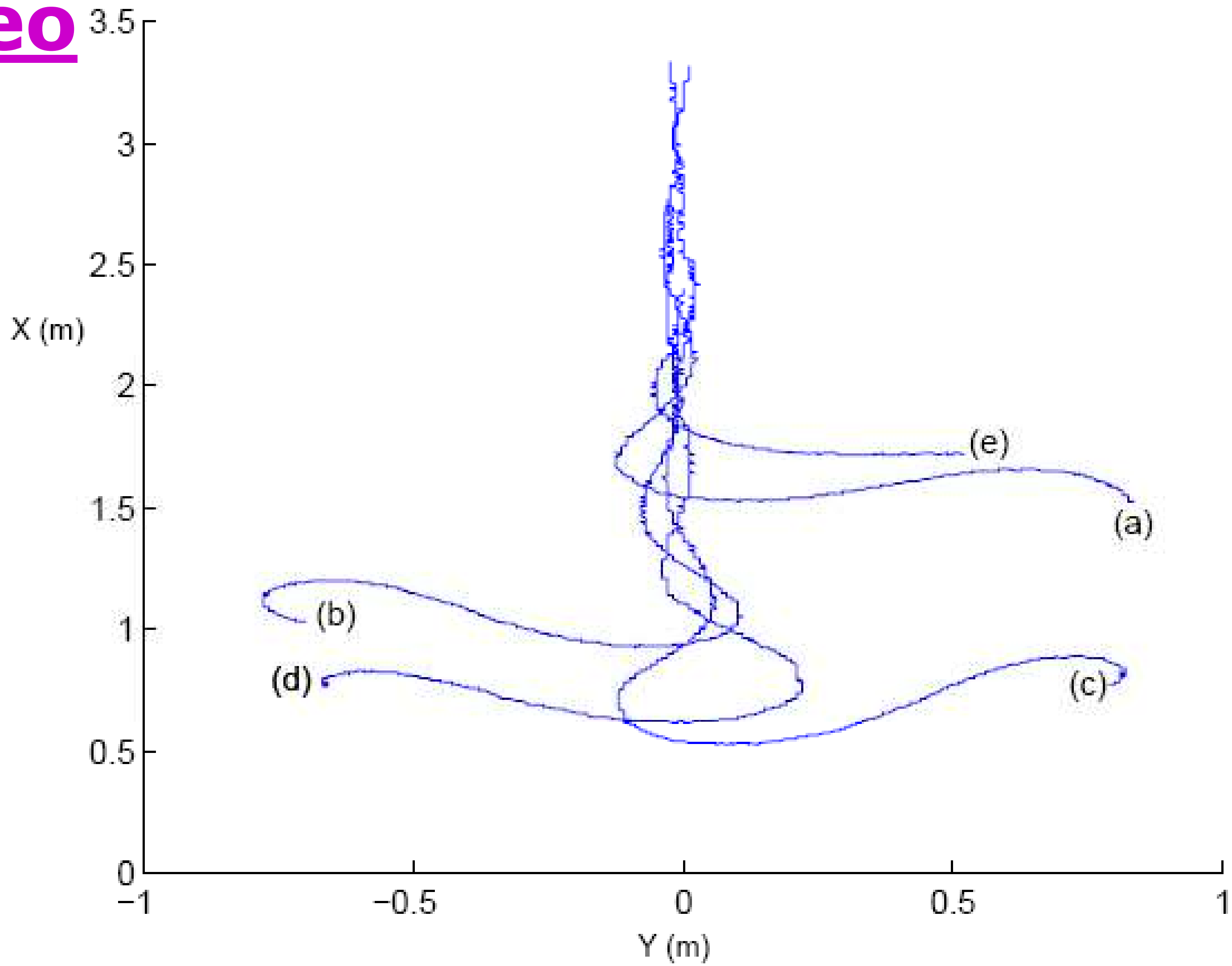




$$x(0) = [\psi,\ r,\ y]^T = [\pi/2,\ 0,\ 0.5]^T$$


Xbee (Arduino)
PWM Control Signal
Xbee (IMU)
Heading Angle
Angular Velocity
Server Computer
X and Y positions
Digital Image Processing
Xbee Wireless Module
Weeled Mobile Robot
Arduino Board
IMU
Lipo Battery

Video



Plot thanks to Behnam Gholitabar

Conclusions

- Applications naturally provide case studies where systems are networked
- Different sensors have different sampling rates
- A theory of stability for multirate networked linear systems was proposed and cast as LMIs
- This theory has been extended to PWA sampled-data systems

Thank you !!! http://hycons.encs.concordia.ca